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Radiative corrections to the Chern–Simons term at finite temperature in the noncommutative Chern–Simons–Higgs model

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Abstract

By analysing the odd parity part of the gauge field two- and three-point vertex functions, the one-loop radiative correction to the Chern–Simons coefficient is computed in the noncommutative Chern–Simons–Higgs model at zero and at high temperature. At high temperature, we show that the static limit of this correction is proportional to T but the first noncommutative correction increases as $T \log T$. Our results are analytic functions of the noncommutative parameter.

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1. Introduction

The analysis of the radiative corrections to the Chern–Simons coefficient has stimulated considerable interest in the recent years [1]. These studies pointed out that the well-known nonrenormalization theorem [2] may become invalid whenever infrared singularities are present. Typical of this possibility are situations which potentially modify the long distance behaviour of the relevant models. Examples are the breakdown of some continuous symmetry, thermal effects and the possible noncommutativity of the underlying space. In this work, we are going to focus on the changes in the Chern–Simons coefficient in a model where all these effects may occur, the noncommutative Chern–Simons–Higgs model.

The appearance of noncommutative coordinates has an old history [3] but gained impetus more recently, mainly due to its connection with string theory [4]. One peculiar aspect of these theories is the ultraviolet/infrared (UV/IR) mixing [5], i.e., the replacement of some ultraviolet divergences by infrared ones. The UV/IR mixing implies the existence of infrared singularities which, at higher orders, may ruin the perturbative expansion. These infrared singularities are generated even in theories without massless fields. However, such behaviour may be ameliorated in some supersymmetric models [6], so that supersymmetry seems to play a decisive role in the construction of consistent noncommutative theories. Up to one loop, the absence of UV/IR mixing has also been verified for the pure $U(n)$ Chern–Simons model which actually seems to be a theory without any quantum correction [7].

In a noncommutative space, trigonometric factors will appear and, because of them, the amplitudes are separated into two parts, the planar and nonplanar ones. Unless by phase factors which depend only on the external momenta, the planar part of the amplitudes is proportional to the corresponding amplitude in the commutative case. The main effects coming from the noncommutativity of the space can be extracted from the nonplanar contributions. These also contain phase factors but, unlike the planar case, they depend on the loop momenta.

Besides the noncommutativity of the space, thermal effects also modify the long distance behaviour of field theories. Aiming to understand the changes induced by thermal effects, many features of noncommutativity at finite temperature have been examined [8]. At finite temperature, it is known that for small momenta the amplitudes in a commutative model are, in general, not well behaved and this feature is understood in terms of the new structure introduced by the temperature, the velocity of the heat bath [9]. On the other hand, in a noncommutative space, the new structure is the $\theta^{\mu\nu}$ tensor, which measures the noncommutativity of the space, and it also leads to infrared nonanalytic behaviour.

Another effect which induces changes in the CS coefficient is the spontaneous breakdown of some continuous symmetry. In the commutative space it is known that classically, in the absence of spontaneous symmetry breakdown and at zero temperature, the non-Abelian coefficient of the Chern–Simons term must be quantized [10]. This was indeed verified first up to one loop in [11] and then extended to all orders in the $SU(\mathrm{N})$ Yang–Mills Chern–Simons model [12]. Similar results hold for noncommutative theories [13]. When spontaneous symmetry breakdown is at work, the quantization condition mentioned above still holds unless no symmetry is left unbroken [14].

In this work, we will study the corrections to the Chern–Simons coefficient at finite temperature arising from the one-loop contributions to the gauge field two- and three-point vertex functions, in the broken phase of the noncommutative Chern–Simons–Higgs model. Unless for some special limits, in noncommutative finite temperature field theory there are many difficulties to evaluate amplitudes in a closed form. Thus, similarly to [15], we will consider a generalization of the hard thermal loop limit, which involves the noncommutative parameter. We then prove that in the static limit and at high temperature, the Chern–Simons term increases like T and actually does not depend on the noncommutative parameter. However, at the next level of approximation, which is linear in the noncommutative parameter, we found that the odd part of the two-point vertex function increases as $T \log T$. As expected, although we are considering the Abelian model, because of the noncommutativity, there will be strong resemblances with the non-Abelian theory.

The work is organized as follows: in section 2, the noncommutative version of the Chern–Simons–Higgs model in the broken phase is presented. Section 3 contains the one-loop corrections to the odd parity part of the gauge field two-point vertex function, both in commutative and in noncommutative cases and at finite temperature in the imaginary time formalism. The zero temperature results are obtained as a consequence of this evaluation. The odd parity part of the gauge field three-point vertex function is studied in section 4. Finally, in section 5 the conclusions are presented. The appendix presents some useful integrals used in the work.

2. Noncommutative Chern–Simons–Higgs model

Noncommutative quantum field theories are defined in a space where the coordinates do not commute among themselves. Rather, the commutator between two position operators is postulated to be

$$[\hat{x}^\mu, \hat{x}^\nu] = \mathrm{i}\theta^{\mu\nu}, \tag{1}$$

where $\theta^{\mu\nu}$ is an antisymmetric matrix, which for simplicity we take as commuting with the $\hat{x}$. The algebra of operators in such space has been extensively studied [17], and many properties are known (see [18] for some reviews). A basic result is that, due to the Wigner–Moyal correspondence, instead of working with functions of the noncommutative coordinates, one may use ordinary functions of commutative variables embodied with the so-called Moyal product, defined as

$$f(x) * g(x) = \left[\mathrm{e}^{(\mathrm{i}/2)\theta^{\mu\nu}\partial_\mu^{(\zeta)}\partial_\nu^{(\eta)}} f(x+\zeta)g(x+\eta)\right]_{\zeta=0=\eta}. \tag{2}$$

Using this definition, one can study quantum field theories in a noncommutative space, by replacing the standard pointwise product of fields by the Moyal one. For simplicity, in this work we shall keep $\theta^{0i} = 0$.

In the present work, we will study the Chern–Simons–Higgs model in a noncommutative space. The model is defined by the action

$$S = \frac{1}{2}\int \mathrm{d}^3x\, \epsilon^{\mu\nu\lambda}\left[A_\mu * \partial_\nu A_\lambda + \frac{2\mathrm{i}g}{3}A_\mu * A_\nu * A_\lambda\right] + (D_\mu\Phi) * (D^\mu\Phi)^\dagger - \frac{\lambda}{4}[\Phi * \Phi^\dagger - v^2]_*^2, \tag{3}$$

where v, g and λ are constants. $D_\mu\Phi$ is the covariant derivative, defined in such way to ensure the gauge invariance of the action. Because of the noncommutativity, under a $U(1)$ gauge transformation U the basic fields may alternatively transform as

(1) fundamental representation: $\Phi \to \Phi * U$ and the covariant derivative is given by $D_\mu\Phi = \partial_\mu\Phi - \mathrm{i}g\Phi * A_\mu$;
(2) anti-fundamental representation: $\Phi \to U^{-1} * \Phi$ and $D_\mu\Phi = \partial_\mu\Phi + \mathrm{i}gA_\mu * \Phi$;
(3) adjoint representation: $\Phi \to U^{-1} * \Phi * U$ and $D_\mu\Phi = \partial_\mu\Phi + \mathrm{i}g[A_\mu, \Phi]_*$. Throughout this paper we will employ the notation $[\,,]_*$ and $\{\,,\}_*$ to respectively designate the commutator and anticommutator using the Moyal product.

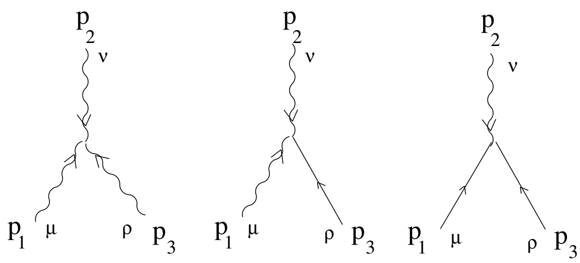
In all theses cases

$$A_\mu \to U^{-1} * A_\mu * U - \frac{1}{\mathrm{i}g}(\partial_\mu U^{-1}) * U. \tag{4}$$

In the adjoint representation, the Higgs mechanism and the induction of new terms containing the gauge fields due the spontaneous breakdown of the gauge symmetry are absent. Because of that we are going to restrict our considerations to the fundamental representation (the analysis of the anti-fundamental representation is actually very similar).

In the spontaneously broken phase, $\langle\Phi\rangle \equiv v \neq 0$, one can choose the decomposition $\Phi = v + \frac{1}{\sqrt{2}}(\sigma + \mathrm{i}\chi)$ and rewrite equation (3) as

$$\begin{aligned} S = \int \mathrm{d}^3x \frac{1}{2}\epsilon^{\mu\nu\lambda}\left(A_\mu * \partial_\nu A_\lambda + \frac{2\mathrm{i}g}{3}A_\mu * A_\nu * A_\lambda\right) + \frac{m}{2}A_\mu * A^\mu - \frac{1}{2\xi}(\partial_\mu A^\mu) * (\partial_\nu A^\nu) \\ + \frac{1}{2}(\partial_\mu\sigma) * (\partial^\mu\sigma) - \frac{m_\sigma^2}{2}\sigma * \sigma + \frac{1}{2}(\partial_\mu\chi) * (\partial^\mu\chi) - \frac{m_\chi^2}{2}\chi * \chi \\ - \frac{g}{2}A_\mu * (\sigma * \overleftrightarrow{\partial^\mu}\chi - \chi * \overleftrightarrow{\partial^\mu}\sigma - \mathrm{i}[\sigma, \partial^\mu\sigma]_* - \mathrm{i}[\chi, \partial^\mu\chi]_*) \end{aligned}$$



(*a*) (*b*) (*c*)

Figure 1. Vertices contributing to the odd parity violating part of the A_μ two- and three-point vertex functions.

$$
\begin{aligned}
&+\frac{g^2}{2}A_\mu * A^\mu * (\sigma * \sigma + \chi * \chi + 2\sqrt{2}v\sigma + \mathrm{i}[\sigma, \chi]_*) \\
&-\frac{\lambda}{2\sqrt{2}}v\left\{\sigma, \frac{(\sigma * \sigma + \chi * \chi)}{2} + \frac{\mathrm{i}}{2}[\chi, \sigma]_*\right\}_* - \frac{\lambda}{16}(\sigma * \sigma + \chi * \sigma + \mathrm{i}[\chi, \sigma]_*)^2_*,
\end{aligned}
\tag{5}
$$

where we have chosen the R_ξ gauge, specified by the gauge fixing action

$$
S_{\mathrm{GF}} = -\frac{1}{2\xi}\int \mathrm{d}^3x(\partial_\mu A^\mu + \xi\sqrt{2}v\chi)^2_*,
\tag{6}
$$

which has the merit of cancelling the nondiagonal terms in the quadratic part of the model. We have also defined

$$
m = 2(gv)^2, \qquad m_\sigma^2 = \lambda v^2, \qquad m_\chi^2 = \xi m.
\tag{7}
$$

To complete the action of the model, one has to add to equation (5) the Faddeev–Popov action given by

$$
S_{\mathrm{FP}} = \int \mathrm{d}^3x[\partial_\mu\bar{c} * \partial^\mu c + \mathrm{i}\partial_\mu\bar{c} * (c * A_\mu - A_\mu * c) + \mathrm{i}\xi v\bar{c} * \chi * c],
\tag{8}
$$

where $\bar{c}$ and c are the ghost fields.

The propagator for the gauge field is

$$
D_{\mu\nu}(p) = \frac{\mathrm{i}}{p^2 - m^2}\left[-mg_{\mu\nu} + p_\mu p_\nu\frac{m-\xi}{p^2+\xi m} + \mathrm{i}\epsilon_{\mu\nu\lambda}p^\lambda\right]
\tag{9}
$$

and for the other fields they are the standard ones ($D_\sigma(p) = \mathrm{i}/(p^2 - m_\sigma^2)$, $D_\chi = \mathrm{i}/(p^2 - m_\chi^2)$ and $D_c = \mathrm{i}/p^2$). These propagators are not affected by the noncommutativity. We will use the following analytic expression for the vertices in figure 1 (we list only those that will contribute in our calculation):

$$
\mathrm{i}A_\mu * A_\nu * A_\rho \qquad \text{vertex} \qquad \leftrightarrow \qquad 2\mathrm{i}g\epsilon_{\mu\rho\nu}\sin(p_1 \wedge p_2)
\tag{10}
$$

$$
A_\mu * A_\nu * \sigma \qquad \text{vertex} \qquad \leftrightarrow \qquad 2\sqrt{2}\mathrm{i}vg^2 g_{\mu\nu}\cos(p_1 \wedge p_2)
\tag{11}
$$

$$
\mathrm{i}A_\rho * [\sigma, \partial^\rho\sigma]_* \qquad \text{vertex} \qquad \leftrightarrow \qquad 2gp_3^\rho\sin(p_2 \wedge p_3).
\tag{12}
$$

At finite temperature and using the imaginary time formalism, the gauge propagator is

$$
D_{\mu\nu}(p) = \frac{1}{p^2 + m^2}\left[m\delta_{\mu\nu} - p_\mu p_\nu\frac{m-\xi}{p^2+\xi m} - \epsilon_{\mu\nu\lambda}p^\lambda\right],
\tag{13}
$$

with $p^\mu \equiv (p^0, \vec{p}) = (2\pi nT, \vec{p})$.

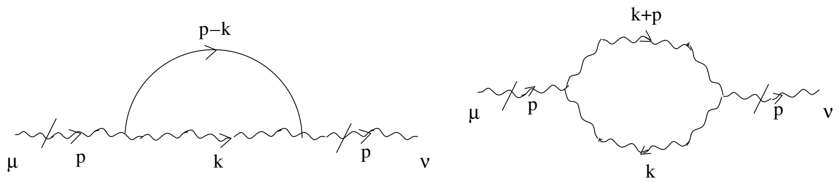


Figure 2. One-loop graphs contributing to the parity violating part of the A_μ two-point vertex function.

3. Two-point function

In this section, we will compute the one-loop radiative correction to the gauge field two-point vertex function of the above model at finite temperature, in both commutative and noncommutative cases, employing the imaginary time formalism. In the noncommutative situation, to fix the behaviour of the Chern–Simons term we will have to examine the corrections to the gauge field two- and three-point vertex functions. We begin by first considering the corrections to the two-point vertex function.

3.1. Commutative case

Let us start by evaluating the parity violating part of the polarization tensor $\Pi_{\mu\nu}$ in the commutative case. There is only one diagram, figure 2(*a*), to evaluate. At one loop, the ghost field does not contribute to the parity violating part, as can be seen from S_{FP}. Thus, we have

$$\Pi^{\mathrm{odd}}_{\mu\nu}(p) \equiv \pi_{\mu\nu}(p) = 8(vg^2)^2 T \sum_n \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{\epsilon_{\mu\nu\lambda}k^\lambda}{(k^2+m^2)\left[(p-k)^2+m_\sigma^2\right]}. \tag{14}$$

The consideration of the static limit, where $p_0 = 0$ and $|\vec{p}|$ is small, yields

$$\pi_{0i}(p_0=0) = 8(vg^2)^2 \int \frac{\mathrm{d}^2k}{(2\pi)^2}\epsilon_{0ij}k^j T\sum_n \frac{1}{k_0^2+w_m^2}\frac{1}{k_0^2+w_\sigma^2(p)}. \tag{15}$$

Note that, in the static limit, π_{ij} vanishes as the integrand is an odd function of n. We have also introduced the notation $w_m^2 = \vec{k}^2 + m^2$ and $w_\sigma^2(p) = (\vec{p}-\vec{k})^2 + m_\sigma^2$. Using that

$$\begin{aligned}\frac{1}{k_0^2+(\vec{p}-\vec{k})^2+m_\sigma^2} &= \frac{1}{k_0^2+w_\sigma^2} + \frac{2\vec{k}\cdot\vec{p}}{\left(k_0^2+w_\sigma^2\right)^2} + \mathcal{O}(\vec{p}^2)\\ &= \left(1-2\vec{k}\cdot\vec{p}\frac{\partial}{\partial m_\sigma^2}\right)\frac{1}{k_0^2+w_\sigma^2} + \mathcal{O}(\vec{p}^2),\end{aligned} \tag{16}$$

where $w_\sigma \equiv w_\sigma(0)$, we write equation (15) for small external momentum as

$$\pi_{0i}(p_0=0) = 8(vg^2)^2\int\frac{\mathrm{d}^2k}{(2\pi)^2}\left(1-2\vec{k}\cdot\vec{p}\frac{\partial}{\partial m_\sigma^2}\right)\epsilon_{0ij}k^jT\sum_n\frac{1}{k_0^2+w_m^2}\frac{1}{k_0^2+w_\sigma^2} + \mathcal{O}(\vec{p}^2). \tag{17}$$

After that, we evaluate the sum in k_0 by using

$$\sum_{n=-\infty}^{n=+\infty} f(n) = -\pi \sum{}' [f(z)\cot(\pi z)], \tag{18}$$

where the prime in the sum indicates that it runs over the residues of the poles of $f(z)$. Thus, we find

$$\pi_{0i}(p_0 = 0) = 4(vg^2)^2 \int \frac{\mathrm{d}^2k}{(2\pi)^2}\epsilon_{0ij}k^j \times \left(1 - 2\vec{k}\cdot\vec{p}\frac{\partial}{\partial m_\sigma^2}\right)\left\{\frac{1}{m_\sigma^2 - m^2}\left[\frac{\coth(\beta w_m/2)}{w_m} - \frac{\coth(\beta w_\sigma/2)}{w_\sigma}\right]\right\}. \quad (19)$$

Using that $\coth(\beta x/2) = 1 + \frac{2}{\mathrm{e}^{\beta x}-1}$, we can separate the zero temperature from the finite temperature part. The zero temperature part can be computed straightforwardly, giving

$$\pi_{0i}(p_0 = 0; T = 0) = 4(vg^2)^2 \int \frac{\mathrm{d}^2k}{(2\pi)^2}\epsilon_{0ij}k^j\left(1 - 2\vec{k}\cdot\vec{p}\frac{\partial}{\partial m_\sigma^2}\right) \times \left[\left(\frac{1}{m_\sigma^2 - m^2}\right)\left(\frac{1}{w_m} - \frac{1}{w_\sigma}\right)\right] = \frac{2g^2}{3\pi}\frac{\left(1 + \frac{1}{2}\frac{m_\sigma}{m}\right)}{\left(1 + \frac{m_\sigma}{m}\right)^2}\epsilon_{0ij}p^j. \quad (20)$$

The finite temperature part in equation (19) is more complicated but can be expressed in terms of polylogarithm functions as

$$\pi_{0i}(p_0 = 0; T) = -\frac{4(vg^2)^2}{\pi}\epsilon_{0ij}p^j\frac{\partial}{\partial m_\sigma^2}\left(\frac{f(m, m_\sigma, T)}{m_\sigma^2 - m^2}\right), \quad (21)$$

where (for properties of polylogarithm functions see [19])

$$f(m, m_\sigma, T) = [T^3\,\mathrm{PolyLog}(3, \mathrm{e}^{-\beta m}) + mT^2\,\mathrm{PolyLog}(2, \mathrm{e}^{-\beta m})] - [m \to m_\sigma] \quad (22)$$

and

$$\mathrm{PolyLog}(b, a) \equiv \sum_{n=1}^{\infty}\frac{a^n}{n^b}. \quad (23)$$

In the high temperature limit, the above expression furnishes

$$\pi_{0i}(p_0 = 0; T) = -\frac{4(vg^2)^2}{\pi}\epsilon_{0ij}p^jT\frac{\partial}{\partial m_\sigma^2}\left[\frac{m_\sigma^2\log(m/m_\sigma)}{m_\sigma^2 - m^2}\right]. \quad (24)$$

The results (20) and (24) agree with the corresponding ones in the Chern–Simons–Higgs limit of the Maxwell–Chern–Simons–Higgs model calculated in [16].

3.2. Noncommutative case

Next, let us determine the parity violating part of the polarization tensor in the noncommutative Chern–Simons–Higgs model. In this case, both diagrams in figure 2 contribute. As in the commutative case, at one loop the ghost field does not give a parity violating part correction to the polarization tensor. The diagram in figure 2(*a*) gives

$$\Pi^{\mathrm{odd}}_{a,\mu\nu} \equiv \pi_{a,\mu\nu}(p) = 8(vg^2)^2T\sum_n\int\frac{\mathrm{d}^2k}{(2\pi)^2}\frac{\epsilon_{\mu\nu\lambda}k^\lambda\cos^2(k\wedge p)}{(k^2+m^2)\left[(p-k)^2+m_\sigma^2\right]}, \quad (25)$$

where $k\wedge p = \frac{1}{2}\theta^{\mu\nu}k_\mu p_\nu = \frac{1}{2}k_\mu\tilde{p}^\mu$ with $\tilde{p}^\mu \equiv \theta^{\mu\nu}p_\nu$. Again, we will consider only the static limit. Therefore, we have

$$\pi_{a,0i}(p_0 = 0) = 4(vg^2)^2\int\frac{\mathrm{d}^2k}{(2\pi)^2}[1 + \cos(2\vec{k}\wedge\vec{p})]\epsilon_{0ij}k^jT\sum_n\frac{1}{k_0^2 + w_m^2}\frac{1}{k_0^2 + w_\sigma^2(p)}. \quad (26)$$

By following the same steps as in the calculation presented in the previous subsection, we arrive at

$$\begin{aligned}\pi_{a,0i}(p_0=0) &= 2(vg^2)^2\int\frac{\mathrm{d}^2k}{(2\pi)^2}[1+\cos(\vec{k}\cdot\tilde{\vec{p}})]\epsilon_{0ij}k^j\\ &\quad\times\left(1-2\vec{k}\cdot\vec{p}\frac{\partial}{\partial m_\sigma^2}\right)\left\{\frac{1}{m_\sigma^2-m^2}\left[\frac{\coth\left(\beta w_m/2\right)}{w_m}-\frac{\coth(\beta w_\sigma/2)}{w_\sigma}\right]\right\}\\ &\equiv A_{0i}+B_{0i},\end{aligned}\tag{27}$$

where A_{0i} and B_{0i} are respectively the planar and nonplanar contributions to $\pi_{a,0i}(p_0=0)$; the $\tilde{p}\equiv|\tilde{\vec{p}}|\to 0$ limit will be taken shortly. The planar part is exactly one-half of equation (20). Thus, considering the nonplanar contribution, from equation (27) we have

$$\begin{aligned}B_{0i} &= 2(vg^2)^2\int\frac{\mathrm{d}^2k}{(2\pi)^2}\cos(\vec{k}\cdot\tilde{\vec{p}})\epsilon_{0ij}k^j\\ &\quad\times\left(1-2\vec{k}\cdot\vec{p}\frac{\partial}{\partial m_\sigma^2}\right)\left\{\frac{1}{m_\sigma^2-m^2}\left[\frac{\coth(\beta w_m/2)}{w_m}-\frac{\coth(\beta w_\sigma/2)}{w_\sigma}\right]\right\}\\ &= -4(vg^2)^2\int\frac{\mathrm{d}^2k}{(2\pi)^2}\cos(\vec{k}\cdot\tilde{\vec{p}})\epsilon_{0ij}k^j\vec{k}\cdot\vec{p}\\ &\quad\times\frac{\partial}{\partial m_\sigma^2}\left\{\frac{1}{m_\sigma^2-m^2}\left[\frac{\coth(\beta w_m/2)}{w_m}-\frac{\coth(\beta w_\sigma/2)}{w_\sigma}\right]\right\}\\ &= -4(vg^2)^2\epsilon_{0ij}p^j\int\frac{\mathrm{d}^2k}{(2\pi)^2}\frac{\cos(\vec{k}\cdot\tilde{\vec{p}})(\vec{k}\cdot\vec{p})^2}{|\vec{p}|^2}\\ &\quad\times\frac{\partial}{\partial m_\sigma^2}\left\{\frac{1}{m_\sigma^2-m^2}\left[\frac{\coth(\beta w_m/2)}{w_m}-\frac{\coth(\beta w_\sigma/2)}{w_\sigma}\right]\right\},\end{aligned}\tag{28}$$

where we have used polar coordinates such that

$$\vec{k}\equiv\frac{(\vec{k}\cdot\vec{p})\vec{p}}{|\vec{p}|^2}+\frac{(\vec{k}\cdot\tilde{\vec{p}})\tilde{\vec{p}}}{|\tilde{\vec{p}}|^2}.\tag{29}$$

The angular part of the above integral can be expressed in terms of Bessel functions, as follows:

$$B_{0i}=-2(vg^2)^2\frac{\epsilon_{0ij}p^j}{\pi}\frac{\partial}{\partial m_\sigma^2}\left\{\frac{1}{m_\sigma^2-m^2}\int_0^\infty\mathrm{d}k\frac{k^2}{\tilde{p}}J_1(k\tilde{p})\left[\frac{\coth(\beta w_m/2)}{w_m}-\frac{\coth(\beta w_\sigma/2)}{w_\sigma}\right]\right\}.\tag{30}$$

These integrals are evaluated in the appendix. Here, we will only write the final results. So, we have

$$B_{0i}(T=0)=-\frac{2(vg^2)^2}{\pi}\epsilon_{0ij}p^j\frac{\partial}{\partial m_\sigma^2}\left\{\frac{1}{m_\sigma^2-m^2}\left[\left(m\frac{\mathrm{e}^{-\tilde{p}m}}{(\tilde{p})^2}+\frac{\mathrm{e}^{-\tilde{p}m}}{(\tilde{p})^3}\right)-(m\to m_\sigma)\right]\right\}\tag{31}$$

and

$$\begin{aligned}B_{0i}(T) &= -\frac{4(vg^2)^2}{\pi}\epsilon_{0ij}p^j\frac{\partial}{\partial m_\sigma^2}\\ &\quad\times\left\{\frac{1}{m_\sigma^2-m^2}\left[\left(mT^2\sum_{n=1}^\infty\frac{\mathrm{e}^{-\beta m\sqrt{n^2+\tau^2}}}{n^2+\tau^2}+T^3\sum_{n=1}^\infty\frac{\mathrm{e}^{-\beta m\sqrt{n^2+\tau^2}}}{(n^2+\tau^2)^{3/2}}\right)-(m\to m_\sigma)\right]\right\},\end{aligned}\tag{32}$$

where $\tau \equiv \tilde{p}T$. Note the apparent singularity in equation (31) at $\tilde{p} = 0$; this kind of structure is the well-known infrared singularity, characteristic of noncommutative field theories [5]. Here, however, as we will soon see, this singularity cancels in the final result. Next, let us check if these results are consistent with the $\theta = 0$ limit, namely, if in this limit we obtain the other half of equations (20) and (24), so that the commutative result is recovered. For the zero temperature part, expanding equation (31) for small values of $\tilde{p}$, we get

$$B_{0i}(T=0) = \epsilon_{0ij}p^j \left[\frac{g^2}{3\pi} \frac{\left(1 + \frac{1}{2}\frac{m_\sigma}{m}\right)}{\left(1 + \frac{m_\sigma}{m}\right)^2} - \frac{(vg^2)^2}{4\pi}\tilde{p} \right] + \mathcal{O}(\tilde{p}^2). \tag{33}$$

When $\theta \to 0$, we obtain one-half of equation (20), completing the expected result for the commutative case.

Now, looking at the temperature-dependent part, equation (32), when $\theta = 0$, we found that the result is again proportional to the function defined in equation (22):

$$B_{0i}(T) = -\frac{4(vg^2)^2}{\pi}\epsilon_{0ij}p^j \frac{\partial}{\partial m_\sigma^2}\left(\frac{f(m, m_\sigma, T)}{m_\sigma^2 - m^2} \right). \tag{34}$$

From the asymptotic behaviour of the polylogarithm functions [19], we found that in the high temperature limit this gives

$$B_{0i}(T) \to \frac{-2(vg^2)^2}{\pi}\epsilon_{0ij}p^j T \frac{\partial}{\partial m_\sigma^2}\left[\frac{m_\sigma^2 \log(m/m_\sigma)}{m_\sigma^2 - m^2} \right] \equiv \frac{1}{2}\pi_{0i}(p_0 = 0, T). \tag{35}$$

Once we have obtained the commutative limit, we consider next the first correction, which in this case is proportional to τ^2. The computations for this term are similar to the ones that we have done so far, and finally, we can write the result, up to τ^2, as

$$\begin{aligned} B_{0i}(T) = -\frac{2(vg^2)^2}{\pi}\epsilon_{0ij}p^j T \frac{\partial}{\partial m_\sigma^2}\Bigg\{ \frac{1}{m_\sigma^2 - m^2}\Bigg[m_\sigma^2 \log(m/m_\sigma) \\ + \tau^2 \frac{1}{8T^2}\left(m^4 \log(m/T) - m_\sigma^4 \log(m_\sigma/T) \right) \Bigg] \Bigg\} + \mathcal{O}(\tau^4). \end{aligned} \tag{36}$$

Before proceeding to the computation of the remaining diagram, it is worth noting that both the temperature-independent and the temperature-dependent parts of B_{0i}, equations (33) and (36), are analytic functions of $\tilde{p}$: no infrared singularities show up.

Evaluating the graph in figure 2(b), we have

$$\Pi_{b,\mu\nu}(p) = 2g^2 T \sum_n \int \frac{\mathrm{d}^3k}{(2\pi)^2}\epsilon_{\mu\alpha\beta}\epsilon_{\sigma\rho\nu}\sin^2(k \wedge p) D^{\alpha\rho}(k+p) D^{\sigma\beta}(k) \tag{37}$$

and considering only the parity violating part of this diagram, we obtain

$$\begin{aligned} \pi_{b,\mu\nu}(p) = 2g^2 T \sum_n \int \frac{\mathrm{d}^3k}{(2\pi)^3} \frac{\sin^2(k \wedge p)\epsilon_{\mu\nu\lambda}}{(k^2 + m^2)[(k+p)^2 + m^2]} \\ \times \left\{ m p^\lambda - (m - \xi) k \cdot (k+p) \left[\frac{k^\lambda}{k^2 + \xi m} - \frac{(k+p)^\lambda}{(k+p)^2 + \xi m} \right] \right\}. \end{aligned} \tag{38}$$

The integrals that appear in this expression are similar to those we have computed before. So, without going into details, for $\vec{p} \to 0$ after taking $p_0 = 0$ the calculation gives

$$\pi_{b,0i}(p_0 = 0; T = 0) = \frac{g^2}{16\pi}(3m - \xi)\tilde{p}\epsilon_{0ij}p^j \tag{39}$$

so that at $T = 0$ we obtain the radiative correction to the odd part of the two-point function

$$\Pi^{\text{odd}}_{0i}(T = 0) = \left[\frac{2g^2}{3\pi}\frac{\left(1+\frac{1}{2}\frac{m_\sigma}{m}\right)}{\left(1+\frac{m_\sigma}{m}\right)^2} + \frac{g^2\tilde{p}}{16\pi}(m-\xi)\right]\epsilon_{0ij}p^j. \tag{40}$$

Actually, the above expression with the replacement of $\epsilon_{0ij}p^j$ by $\epsilon_{\mu\nu\rho}p^\rho$ holds also for $\pi_{\mu\nu}$.

On the other hand, for high temperature and also small $\vec{p}$ and τ we have

$$\pi_{b,0i}(p_0 = 0; T) = -\frac{\epsilon_{0ij}p^j}{16\pi}\frac{g^2\tau^2}{T}[2m\log(m/T) + (m-\xi)F], \tag{41}$$

where

$$F \equiv \frac{\xi^2(\xi+m)}{(\xi-m)^3}\log(\sqrt{\xi m}/T) - \frac{m(m^2+4\xi^2-3\xi m)}{(\xi-m)^3}\log(m/T). \tag{42}$$

The complete result for the two-point function at high T in this limit is

$$\begin{aligned}\pi^{\text{NC}}_{0i}(p_0 = 0; T) = {} & -\frac{1}{\pi}\epsilon_{0ij}p^jT \\ & \times\left\{2(v\mathrm{e}^2)^2\frac{\partial}{\partial m_\sigma^2}\left[\frac{2m_\sigma^2\log(m/m_\sigma) + \frac{\tilde{p}^2}{8}\left(m^4\log(m/T) - m_\sigma^4\log(m_\sigma/T)\right)}{m_\sigma^2 - m^2}\right]\right. \\ & \left. + \frac{g^2\tilde{p}^2}{16}\left[2m\log(m/T) + (m-\xi)F\right]\right\}.\end{aligned} \tag{43}$$

It is worth noting here the gauge dependence of π_b. Naively, we could expect that there would be no gauge dependence at this point of the calculation, as happens in the commutative case. But this graph gives a purely noncommutative contribution to $\Pi_{\mu\nu}$, which will disappear when θ goes to zero. Therefore, there is no relation between it and what is found in the commutative case. We recall that a dependence on the gauge parameter was also obtained in the commutative studies of the non-Abelian gauge models [11].

4. Three-point function

To complete our analysis on the radiative corrections to the Chern–Simons term, we will now compute the odd parity part of the one-loop contribution to the gauge field three-point vertex function. For simplicity, we shall calculate only the small momenta leading corrections. As we will see, this implies that the contributions in which we are interested are proportional to the product of the Levi-Cività symbol by a trigonometric sine factor.

To extract the $T = 0$ contribution, we will use the identity [20]

$$\frac{\mathrm{i}}{\beta}\sum_n f\left(\vec{k}, k_0 = \frac{2\pi n}{\beta}\right) = \int_{-\mathrm{i}\infty+\epsilon}^{\mathrm{i}\infty+\epsilon}\mathrm{d}k_0\frac{f(\vec{k},k_0)}{\mathrm{e}^{\beta k_0}-1} + \int_{-\mathrm{i}\infty-\epsilon}^{\mathrm{i}\infty-\epsilon}\mathrm{d}k_0\frac{f(\vec{k},k_0)}{\mathrm{e}^{-\beta k_0}-1} + \int_{-\mathrm{i}\infty}^{\mathrm{i}\infty}\mathrm{d}k_0 f(\vec{k},k_0) \tag{44}$$

and employ the more compact notation

$$\int_{T=0}\frac{\mathrm{d}^3k}{(2\pi)^3} \equiv \int\frac{\mathrm{d}^2k}{(2\pi)^2}\int_{-\mathrm{i}\infty}^{\mathrm{i}\infty}\frac{\mathrm{d}k_0}{(2\pi)}, \tag{45}$$

$$\int_{T\neq 0}\frac{\mathrm{d}^3k}{(2\pi)^3} \equiv \int\frac{\mathrm{d}^2k}{(2\pi)^2}\left[\int_{-\mathrm{i}\infty+\epsilon}^{\mathrm{i}\infty+\epsilon}\frac{\mathrm{d}k_0}{(2\pi)}\frac{1}{\mathrm{e}^{\beta k_0}-1} + \int_{-\mathrm{i}\infty-\epsilon}^{\mathrm{i}\infty-\epsilon}\frac{\mathrm{d}k_0}{(2\pi)}\frac{1}{\mathrm{e}^{-\beta k_0}-1}\right]. \tag{46}$$

The relevant graphs are drawn in figure 3. Here we will present our results for zero and nonzero temperature.

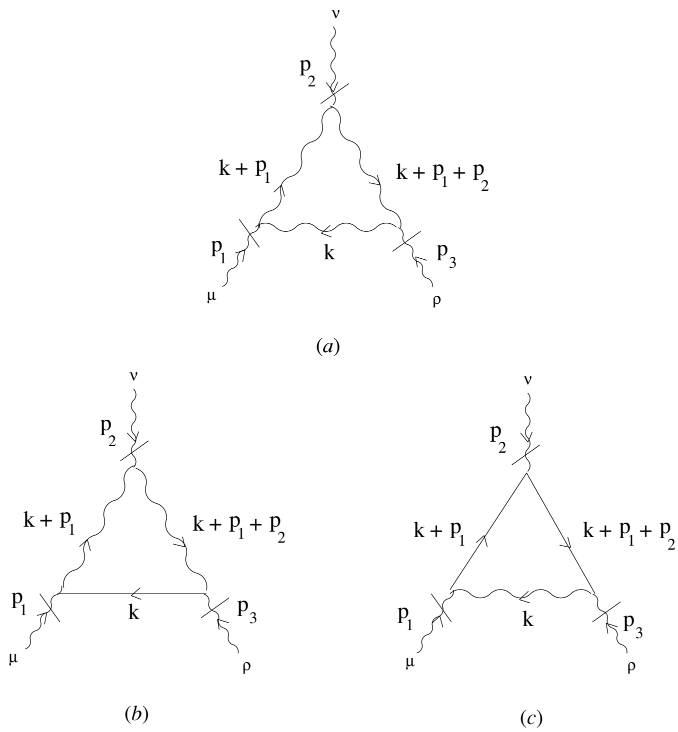


Figure 3. One-loop graphs contributing to the parity violating part of the A_μ three-point vertex function.

4.1. $T = 0$ results

Using the expressions for the propagators and vertices listed in the previous section, the amplitudes for the graphs which contribute to the odd parity part of the three-point vertex function of the gauge field are

(1) Graph in figure 3(*a*)

$$\Gamma^{3a}_{\mu\nu\rho} = -8g^3 \int_{T=0} \frac{\mathrm{d}^3k}{(2\pi)^3} \mathcal{D}^{\sigma\alpha}(k+p_1)\mathcal{D}^{\beta\tau}(k-p_3)\mathcal{D}^{\xi\lambda}(k)\epsilon_{\alpha\nu\beta}\epsilon_{\lambda\mu\sigma}\epsilon_{\tau\rho\xi}S_1, \tag{47}$$

where

$$\begin{aligned} S_1 &= \sin(k\wedge p_1)\sin(k\wedge p_3)\sin[(k+p_1)\wedge p_2] \\ &= -\tfrac{1}{4}\{\sin(p_1\wedge p_2) + \sin[(2k+p_2)\wedge p_1] + \sin[(2k+p_1)\wedge p_2] \\ &\quad + \sin[(2k+p_1)\wedge p_3]\}. \end{aligned} \tag{48}$$

(2) Graph in figure 3(*b*)

$$\Gamma^{3b}_{\mu\nu\rho} = \frac{-112\mathrm{i}v^2g^5}{3} \int_{T=0} \frac{\mathrm{d}^3k}{(2\pi)^3} \mathcal{D}_{\mu\alpha}(k+p_1)\mathcal{D}_{\beta\rho}(k-p_3)\Delta_\sigma(k)\epsilon_\nu{}^{\beta\alpha}S_2, \tag{49}$$

where

$$\begin{aligned} S_2 &= \cos(k\wedge p_1)\cos(k\wedge p_3)\sin[(k+p_1)\wedge p_2] \\ &= \tfrac{1}{4}\{\sin(p_1\wedge p_2) - \sin[(2k+p_2)\wedge p_1] + \sin[(2k+p_1)\wedge p_2] - \sin[(2k+p_1)\wedge p_3]\}. \end{aligned} \tag{50}$$

(3) Graph in figure 3(c)

$$\Gamma^{3c}_{\mu\nu\rho} = -80v^2 g^5 \int_{T=0} \frac{d^3k}{(2\pi)^3} (p_2+k)_\nu \mathcal{D}_{\rho\mu}(k)\Delta_\sigma(k+p_1)\Delta_\sigma(k-p_3)S_3, \tag{51}$$

where

$$\begin{aligned} S_3 &= \cos(k\wedge p_2)\cos(k\wedge p_3)\sin[(k+p_2)\wedge p_1] \\ &= \tfrac{1}{4}\{\sin(p_2\wedge p_1) - \sin[(2k+p_1)\wedge p_2] + \sin[(2k+p_2)\wedge p_1] - \sin[(2k+p_2)\wedge p_3]\}. \end{aligned} \tag{52}$$

In the planar parts, which are the integrals containing the first term in S_1, S_2 and S_3, we separate the trigonometric factor and set equal to zero the external momenta in the propagators. In the remaining (nonplanar) parts, we approximate the sines by their arguments and then perform the integrals.

It turns out that within our approximation, $\Gamma^{3a}_{\mu\nu\rho}$ does not possess an odd parity part. In fact, because of momentum conservation it is readily seen that S_1 becomes null if the sines are replaced by their arguments. We then consider separately the odd parity part of the other contributions. We have

$$\begin{aligned} \left[\Gamma^{3b}_{\mu\nu\rho}\right]_{\text{odd}} = \frac{-112v^2g^5}{3} \int_{T=0} \frac{d^3k}{(2\pi)^3} \frac{S_2}{[(k+p_1)^2-m^2][(k-p_3)^2-m^2]\left[k^2-m_\sigma^2\right]} \\ \times \left\{-m^2\epsilon_{\mu\rho\nu} + m^2\left[\frac{\epsilon_{\rho\nu\alpha}k^\alpha k_\mu}{[(k+p_1)^2+\xi m]} - \frac{\epsilon_{\mu\nu\alpha}k^\alpha k_\rho}{[(k-p_3)^2+\xi m]}\right] - \epsilon_{\mu\rho\alpha}k^\alpha k_\nu\right\}, \end{aligned} \tag{53}$$

for the graph 3(b) and

$$\left[\Gamma^{3c}_{\mu\nu\rho}\right]_{\text{odd}} = -80v^2g^5 \int_{T=0} \frac{d^3k}{(2\pi)^3} \frac{\epsilon_{\rho\alpha\mu}k^\alpha k_\nu}{\left[(k+p_1)^2-m_\sigma^2\right]\left[(k-p_3)^2-m_\sigma^2\right][k^2-m^2]} S_3, \tag{54}$$

for the graph 3(c). Proceeding now in the above-indicated way we get

$$\left[\Gamma^{3b}_{\mu\nu\rho}\right]_{\text{odd}} = \frac{-112v^2g^5}{3}\left\{\left[\Gamma^{3b}_{\mu\nu\rho}\right]^{\text{planar}}_{\text{odd}} + \left[X^{3b}_{\mu\nu\rho}\right]_{\text{odd}}\right\}, \tag{55}$$

where

$$\begin{aligned} \left[\Gamma^{3b}_{\mu\nu\rho}\right]^{\text{planar}}_{\text{odd}} &= \sin(p_1\wedge p_2)\int_{T=0} \frac{d^3k}{(2\pi)^3} \frac{1}{[k^2-m^2]^2\left[k^2-m_\sigma^2\right]} \\ &\quad\times\left\{-m^2\epsilon_{\mu\rho\nu} + m^2\left[\frac{\epsilon_{\rho\nu\alpha}k^\alpha k_\mu}{[k^2+\xi m]} - \frac{\epsilon_{\mu\nu\alpha}k^\alpha k_\rho}{[k^2+\xi m]}\right] - \epsilon_{\mu\rho\alpha}k^\alpha k_\nu\right\} \\ &= \left[\frac{-m_\sigma}{12\pi(m+m_\sigma)^2}\right]\sin(p_1\wedge p_2)\epsilon_{\mu\rho\nu} \end{aligned} \tag{56}$$

and

$$\begin{aligned} \left[X^{3b}_{\mu\nu\rho}\right]_{\text{odd}} &= \int_{T=0} \frac{d^3k}{(2\pi)^3} \frac{(-\sin(2k+p_2)\wedge p_1 + \sin(2k+p_1)\wedge p_2 - \sin(2k+p_1)\wedge p_3)}{[(k+p_1)^2-m^2][(k-p_3)^2-m^2]\left[k^2-m_\sigma^2\right]} \\ &\quad\times\left\{-m^2\epsilon_{\mu\rho\nu} + m^2\left[\frac{\epsilon_{\rho\nu\alpha}k^\alpha k_\mu}{[(k+p_1)^2+\xi m]} - \frac{\epsilon_{\mu\nu\alpha}k^\alpha k_\rho}{[(k-p_3)^2+\xi m]}\right] - \epsilon_{\mu\rho\alpha}k^\alpha k_\nu\right\} \\ &= \left[\frac{m^2+mm_\sigma+2m_\sigma^2}{30\pi(m+m_\sigma)^3}\right]\sin(p_1\wedge p_2)\epsilon_{\mu\rho\nu}. \end{aligned} \tag{57}$$

By summing (56) and (57), we obtain the total contribution from the graph 3(*b*)

$$\left[\Gamma^{3b}_{\mu\nu\rho}\right]_{\text{odd}} = \frac{28\mathrm{i}g^5 v^2}{45\pi}\left[\frac{\left(m_\sigma^2 - 2m^2 + 3mm_\sigma\right)}{(m+m_\sigma)^3}\right]\epsilon_{\mu\nu\rho}\sin(p_1\wedge p_2). \tag{58}$$

The computation for $\left[\Gamma^{3c}_{\mu\nu\rho}\right]_{\text{odd}}$ follows similarly yielding

$$\left[\Gamma^{3c}_{\mu\nu\rho}\right]_{\text{odd}} = \frac{\mathrm{i}g^5 v^2}{3\pi}\left[\frac{13m_\sigma^2 + 39mm_\sigma + 28m^2}{(m+m_\sigma)^3}\right]\epsilon_{\mu\nu\rho}\sin(p_1\wedge p_2), \tag{59}$$

so that the complete result for the one loop three-point vertex function correction at $T=0$ is given by

$$\begin{aligned}[\Gamma_{\mu\nu\rho}]_{\text{odd}} &= \left[\Gamma^{3b}_{\mu\nu\rho}\right]_{\text{odd}} + \left[\Gamma^{3c}_{\mu\nu\rho}\right]_{\text{odd}}\\ &= -\frac{\mathrm{i}v^2 g^5}{45\pi(m+m_\sigma)^3}\sin(p_1\wedge p_2)\epsilon_{\mu\rho\nu}\left[476m^2 + 501mm_\sigma + 167m_\sigma^2\right].\end{aligned} \tag{60}$$

4.2. Results for $T \neq 0$

The amplitudes for $T\neq 0$ are associated with the same graphs as in the previous section but using equation (46) in the corresponding analytic expressions. With the help of Feynman parametric integrals, the propagators are combined into a sum of terms with only one factor in the denominator. The k_0 integral is then performed by adequately closing the contour of integration and applying the residue theorem. Afterwards the $\vec{k}$ integrals are calculated in the static limit. We find

1. $T\neq 0$ part of the graph 3(*b*)

$$\begin{aligned}\left[\Gamma^{3b}_{\mu\nu\rho}\right]^T_{\text{odd}} &= \frac{-112v^2g^5}{3}\sin(p_1\wedge p_2)\\ &\quad\times\left\{-2\int_0^1\mathrm{d}x\int_{T\neq 0}\frac{\mathrm{d}^3k}{(2\pi)^3}(1-x)x\left[\frac{m^2\epsilon_{\mu\rho\nu}+\epsilon_{\mu\rho\alpha}k^\alpha k_\nu}{\left(k^2-\Lambda_1^2\right)^3}\right]\right.\\ &\quad\left.+6m^2\int_0^1\mathrm{d}x\int_0^1\mathrm{d}y\int_{T\neq 0}\frac{\mathrm{d}^3k}{(2\pi)^3}[2-y(1-x)]xy^2\left[\frac{\epsilon_{\mu\alpha\nu}k^\alpha k_\rho+\epsilon_{\mu\rho\alpha}k^\alpha k_\nu}{\left(k^2-\Lambda_2^2\right)^4}\right]\right\},\end{aligned} \tag{61}$$

where $\Lambda_1^2 = m_\sigma^2(1-x)+m^2x$ and $\Lambda_2^2 = y\Lambda_1^2$, whereas, for graph 3(*c*)

$$\left[\Gamma^{3c}_{\mu\nu\rho}\right]^T_{\text{odd}} = -80v^2g^5\sin(p_1\wedge p_2)\int_0^1\mathrm{d}x\,x\left(1+\frac{x}{2}\right)\int_{T\neq 0}\frac{\mathrm{d}^3k}{(2\pi)^3}\frac{\epsilon_{\mu\rho\alpha}k^\alpha k_\nu}{\left(k^2-\Lambda_3^2\right)^3}, \tag{62}$$

where $\Lambda_3^2 = m^2(1-x)+m_\sigma^2 x$.

As a prototype for the calculation of the above expression, we consider the typical integral

$$\begin{aligned}I_{\mu\rho\nu} &= \int_{T\neq 0}\frac{\mathrm{d}^3k}{(2\pi)^3}\frac{\epsilon_{\mu\rho\alpha}k^\alpha k_\nu}{(k^2-\Lambda^2)^3}\\ &= \int_{T\neq 0}\frac{\mathrm{d}^3k}{(2\pi)^3}\left[\frac{g_{0\nu}\epsilon_{\mu\rho 0}k_0^2 + g_{\nu j}\epsilon_{\mu\rho i}k^ik^j}{(k^2-\Lambda^2)^3}\right],\end{aligned} \tag{63}$$

where $\Lambda=\Lambda_i$ with $i=1,2,3$. After integrating over k_0 we obtain

$$I_{\mu\rho\nu} = -2\mathrm{i}\int\frac{\mathrm{d}^2k}{(2\pi)^2}[g_{0\nu}\epsilon_{\mu\rho 0}F_1(\vec{k}^2) + g_{\nu i}\epsilon_{\mu\rho i}F_2(\vec{k}^2)], \tag{64}$$

where

$$F_1(\vec{k}^2) = \frac{-1 + e^{2\beta\omega_\Lambda}\left(-1 - \beta\omega_\Lambda + \beta^2\omega_\Lambda^2\right) + e^{\beta\omega_\Lambda}\left(2 + \beta\omega_\Lambda + \beta^2\omega_\Lambda^2\right)}{16\omega_\Lambda^3(e^{\beta\omega_\Lambda} - 1)^3}$$
$$\overset{\beta\to 0}{\simeq} \frac{1}{32\omega_\Lambda^3} + \mathcal{O}(\beta^3) \tag{65}$$

and

$$F_2(\vec{k}^2) = \frac{3 + e^{2\beta\omega_\Lambda}\left(3 + 3\beta\omega_\Lambda + \beta^2\omega_\Lambda^2\right) + e^{\beta\omega_\Lambda}\left(-6 - 3\beta\omega_\Lambda + \beta^2\omega_\Lambda^2\right)}{16\omega_\Lambda^5(e^{\beta\omega_\Lambda} - 1)^3}$$
$$\overset{\beta\to 0}{\simeq} \frac{1}{2\omega_\Lambda^6\beta} - \frac{3}{32\omega_\Lambda^5} + \mathcal{O}(\beta^4) \tag{66}$$

The expressions on the right-hand side of equations (65) and (66) correspond to the high T limit of F_1 and F_2, respectively. In this limit the integrals on the spatial components of k are very simple, giving

$$I_{\mu\rho\nu} = \frac{-i}{16\pi}\left[\frac{1}{\Lambda}\epsilon_{\mu\rho\nu} - \frac{2}{\beta\Lambda^2}g_\nu^i\epsilon_{\mu\rho i}\right] + \mathcal{O}(\beta^3). \tag{67}$$

It remains to perform the Feynman parametric integrals which for expressions like (67) are trivial.

Proceeding as in the above example, the relevant integrals may be straightforwardly computed. We list the results for each contributing graph

$$\left[\Gamma^{3b}_{\mu\nu\rho}\right]^T_{\text{odd}} = \frac{-iv^2g^5\sin(p_1\wedge p_2)}{\pi}\left\{\frac{1}{21\beta}\left[A_1\left(4\epsilon_{\mu\rho\nu} - g_\mu^i\epsilon_{i\rho\nu} - g_\rho^i\epsilon_{\mu i\nu}\right)\right.\right.$$
$$\left.\left. + A_2 g_\nu^i\epsilon_{\mu\rho i}\right] + A_3\epsilon_{\mu\rho\nu}\right\}, \tag{68}$$

where

$$A_1 = 2\frac{\left[m_\sigma^2m^2 - m^4 + \left(m^4 + m^2m_\sigma^2\right)\ln\left(\frac{m}{m_\sigma}\right)\right]}{\left(m^2 - m_\sigma^2\right)^3}, \tag{69}$$

$$A_2 = \frac{m_\sigma^4 - m^4 + 4m^2m_\sigma^2\ln\left(\frac{m}{m_\sigma}\right)}{\left(m^2 - m_\sigma^2\right)^3}, \tag{70}$$

$$A_3 = \frac{28}{45(m + m_\sigma)^3}\left(m_\sigma^2 - 2m^2 + 3mm_\sigma\right), \tag{71}$$

and

$$\left[\Gamma^{3c}_{\mu\nu\rho}\right]^T_{\text{odd}} = \frac{-80iv^2g^5\sin(p_1\wedge p_2)}{\pi}\left[\frac{B_1}{\beta}g_\nu^i\epsilon_{\mu\rho i} + B_2\epsilon_{\mu\nu\rho}\right], \tag{72}$$

where

$$B_1 = \frac{1}{32\left(m^2 - m_\sigma^2\right)^3}\left[5m_\sigma^4 - 12m^2m_\sigma^2 + 7m^4 - 4\left(2m^2m_\sigma^2 - 3m^4\right)\ln\left(\frac{m_\sigma}{m}\right)\right] \tag{73}$$

and

$$B_2 = \frac{1}{240(m + m_\sigma)^3}\left(13m_\sigma^2 + 39mm_\sigma + 28m^2\right). \tag{74}$$

Note that the terms containing A_3 and B_2 coincide up to a sign with the expressions in equations (58) and (59), respectively, so that when computing the high temperature limit of the three-point vertex function they mutually cancel.

5. Conclusions

We can now summarize the results obtained in the previous sections.

(1) *Zero temperature*. For small momenta, the corrections to the two- and three-point vertex functions given in equations (40) and (60) lead to the conclusion that the following action is induced (see the remark after equation (40)):

$$S_{\text{ind}}^{T=0} = \frac{\bar{\kappa}_1}{2} \int \mathrm{d}^3 x \, \epsilon_{\mu\nu\rho} \left[A^\mu \partial^\nu A^\rho + \frac{2\mathrm{i}\bar{g}_1}{3} A^\mu * A^\nu * A^\rho \right], \tag{75}$$

where

$$\bar{\kappa}_1 = \frac{2g^4 v^2}{3\pi} \frac{(2m + m_\sigma)}{(m + m_\sigma)^2} \tag{76}$$

and

$$\bar{g}_1 = g \frac{236m^2 + 231 m m_\sigma + 77 m_\sigma^2}{60(m + m_\sigma)(2m + m_\sigma)}. \tag{77}$$

In the limit $m_\sigma = m$ which is relevant for the supersymmetric Chern–Simons–Higgs model [21], a great simplification is achieved so that $\bar{\kappa}_1 = \frac{g^2}{4\pi}$ and $\bar{g}_1 = \frac{68}{45} g$.

(2) *High temperature limit*.

$$S_{\text{ind}}^T \overset{\beta\to 0}{\simeq} \frac{\bar{\kappa}_2}{2\beta} \int \mathrm{d}^3 x \, \epsilon_{0ij} \left[A^0 \partial^i A^j + \frac{2\mathrm{i}\bar{g}_2}{3} A^0 * A^i * A^j \right], \tag{78}$$

where

$$\bar{g}_2 = \frac{g}{168\left(m^2 - m_\sigma^2\right)} \left[\frac{721 m^4 - 1248 m^2 m_\sigma^2 + 527 m_\sigma^4 + \left(-1248 m^4 + 860 m^2 m_\sigma^2\right) \ln\left(\frac{m}{m_\sigma}\right)}{m_\sigma^2 - m^2 + 2m^2 \ln\left(\frac{m}{m_\sigma}\right)} \right] \tag{79}$$

and

$$\bar{\kappa}_2 = \frac{2v^2 g^4}{\pi \left(m^2 - m_\sigma^2\right)^2} \left[m_\sigma^2 - m^2 + 2m^2 \ln\left(\frac{m}{m_\sigma}\right) \right]. \tag{80}$$

Here again a great simplification occurs for equal masses: $\bar{\kappa}_2 = \frac{1}{4\pi v^2}$ and $\bar{g}_2 = \frac{-839g}{252}$.

The leading noncommutative corrections to the two-point vertex function were also obtained and are given by

$$\Pi_{0i}^{\text{odd}}(T = 0) = \frac{g^2 \tilde{p}}{16\pi} (m - \xi) \epsilon_{0ij} p^j, \tag{81}$$

at zero temperature, and

$$\pi_{0i}^{\text{NC}}(p_0 = 0; T) = -\frac{\tilde{p}^2}{8\pi} \epsilon_{0ij} p^j T \left\{ 2(v\mathrm{e}^2)^2 \frac{\partial}{\partial m_\sigma^2} \left[\frac{\left(m^4 \log(m/T) - m_\sigma^4 \log(m_\sigma/T)\right)}{m_\sigma^2 - m^2} \right] + \frac{g^2}{2} \left[2m \log(m/T) + (m - \xi) F \right] \right\}, \tag{82}$$

in the high temperature limit. From equations (81) and (82) we can see that the commutativity can be recovered straightforwardly, by considering the limit $\tilde{p} \to 0$. In other words, there are no infrared UV/IR singularities appearing in this limit. Furthermore, looking at the finite temperature result, equation (82), in the static limit we can extract the leading behaviour in the high temperature regime and first-order correction in the noncommutative parameter as being proportional to $T \log T$. So, our calculation provides a logarithm correction to the result obtained in [16] for the commutative version of the same model.

The results in equations (75) and (78) are formally invariant under small gauge transformations. Note however that, because of the spontaneous breakdown of the symmetry, such a property will certainly be lost whenever other corrections are incorporated. This is already indicated by the form of the noncommutative corrections in equations (81) and (82).

It should be pointed out that previous studies of noncommutative gauge theories have shown that, as happens in the commutative non-Abelian CS model, invariance under large gauge transformation requires that the CS coefficient be quantized [13]. At finite temperature, the verification of this property for the effective action is a highly non-trivial task which probably involves all orders of perturbation and also other (nonlocal) interactions. Even in the commutative setting, invariance under large non-Abelian gauge transformation was verified only in simplified situations [22]. In our calculations, invariance under large gauge transformation is certainly partially broken. In spite of that, our results should still be a good approximation for small couplings.

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Appendix. An useful integral

In this appendix, we will compute a basic integral that appears in equation (30). Let us define

$$\begin{aligned} I(m) &\equiv \int_0^\infty k^2\,\mathrm{d}k\,J_1(k\tilde{p})\frac{\coth(\beta w_m/2)}{w_m}. \\ &= \int_0^\infty k^2\,\mathrm{d}k\,J_1(k\tilde{p})\frac{1}{w_m} + 2\int_0^\infty k^2\,\mathrm{d}k\,J_1(k\tilde{p})\frac{n_B(w_m)}{w_m}. \end{aligned} \tag{A1}$$

Using that

$$\frac{1}{\mathrm{e}^x - 1} = \sum_{n=1}^\infty \mathrm{e}^{-nx} \tag{A2}$$

we can rewrite $I(m)$ as

$$\begin{aligned} I(m) = &\int_m^\infty \mathrm{d}x\sqrt{x^2 - m^2}\,J_1\big[\tilde{p}\sqrt{x^2 - m^2}\big] \\ &+ T^2\sum_0^\infty \int_{\beta m}^\infty \mathrm{d}x\sqrt{x^2 - \beta m^2}\,J_1\big[\tau\sqrt{x^2 - b^2 m^2}\big]\,\mathrm{e}^{-nx} \\ = &\, m^2\int_1^\infty \mathrm{d}y\sqrt{y^2 - 1}\,J_1\big[\tilde{p}m\sqrt{y^2 - 1}\big] \\ &+ m^2\sum_0^\infty \int_1^\infty \mathrm{d}y\sqrt{y^2 - 1}\,J_1\big[\tau\beta m\sqrt{y^2 - 1}\big]\,\mathrm{e}^{-\beta m y n} \end{aligned} \tag{A3}$$

These integrals can be evaluated using the standard result [23]

$$\int_1^\infty \mathrm{d}x\,(x^2 - 1)^{\nu/2}\,\mathrm{e}^{-\alpha x}\,J_\nu\big[\beta\sqrt{x^2 - 1}\big] = \sqrt{\frac{2}{\pi}}\beta^\nu(\alpha^2 + \beta^2)^{-\nu/2 - 1/4}K_{\nu+1/2}\big(\sqrt{\alpha^2 + \beta^2}\big). \tag{A4}$$

Then, it is straightforward to verify that

$$I(m) = m^2\,\mathrm{e}^{-\tilde{p}m}\left[\frac{1}{\tilde{p}m}+\frac{1}{(\tilde{p}m)^2}\right]+\tilde{p}mT^2\sum_{n=1}^{\infty}\frac{\mathrm{e}^{-\beta m\sqrt{n^2+\tau^2}}}{n^2+\tau^2}+\tilde{p}T^3\sum_{n=1}^{\infty}\frac{\mathrm{e}^{-\beta m\sqrt{n^2+\tau^2}}}{(n^2+\tau^2)^{3/2}}. \qquad \text{(A5)}$$

Note that here we have computed both zero temperature and finite temperature parts together, although in sections 2 and 3 they occur separately.

References

[1] Dunne G V 1999 Aspects of Chern Simons theory *Les Houches Summer School in Theoretical Physics: Topological Aspects of Low Dimensional Physics (Les Houches 1968)*

[2] Coleman S and Hill B 1985 *Phys. Lett.* B **159** 184

[3] Jackiw R 2002 *Nucl. Phys. Proc. Suppl.* **108** 30

[4] Seiberg N and Witten E 1999 *J. High Energy Phys.* JHEP09(1999)032

[5] Minwalla S, Van Raamsdonk M and Seiberg N 2000 *J. High Energy Phys.* JHEP08(2000)008

[6] Girotti H O, Gomes M, Rivelles V O and da Silva A J 2000 *Nucl. Phys.* B **587** 299
Girotti H O, Gomes M, Petrov A Yu, Rivelles V O and da Silva A J 2001 *Phys. Lett.* B **521** 119

[7] Bichl A, Grimstrup J, Putz V and Schweda M 2000 *J. High Energy Phys.* JHEP07(2000)046
Das A and Sheikh-Jabbari M M 2001 *J. High Energy Phys.* JHEP06(2001)028
Martin C P 2001 *Phys. Lett.* B **515** 185

[8] Fischler W, Gommis J, Gorbatov E, Kashani-Poor A, Paban S and Pouliot P 2000 *J. High Energy Phys.* JHEP05(2000)024
Landsteiner K, Lopez E and Tytgat M H G 2000 *J. High Energy Phys.* JHEP09(2000)027
Arcioni G and Vázquez-Mozo M A 2000 *J. High Energy Phys.* JHEP01(2000)028
Brandt F T, Das A, Frenkel J, McKeon D G C and Taylor J C 2002 *Phys. Rev.* D **66** 045011
Chandrasekhar B and Panigrahi P K 2003 *J. High Energy Phys.* JHEP03(2003)015

[9] Kapusta J I 1979 *Nucl. Phys.* B **148** 461
Arnold P, Vokos S, Bedaque P and Das A 1993 *Phys. Rev.* D **47** 4698

[10] Deser S, Jackiw R and Templeton S 1982 *Ann. Phys., NY* **140** 372
Deser S, Jackiw R and Templeton S 1988 *Ann. Phys., NY* **185** 406 (erratum)
Deser S, Jackiw R and Templeton S 2000 *Ann. Phys., NY* **281** 409
Redlich A 1984 *Phys. Rev.* D **29** 2366
Redlich A 1984 *Phys. Rev. Lett.* **52** 18

[11] Pisarski R and Rao S 1985 *Phys. Rev.* D **32** 2081

[12] Brandt F, Das A and Frenkel J 2000 *Phys. Lett.* B **494** 339

[13] Nair V P and Polychronakos A P 2001 *Phys. Rev. Lett.* **87** 030403
Sheikh-Jabbari M M 2001 *Phys. Lett.* B **510** 247
Bak D, Lee K and Park J H 2001 *Phys. Rev. Lett.* **87** 030402
Chen Guang-Hong and Wu Yong-Shi 2001 *Nucl. Phys.* B **593** 562

[14] Khlebinikov S and Shaposhnikov M 1991 *Phys. Lett.* B **254** 148
Khare A, MacKenzie R, Paranjape M and Panigrahi P 1995 *Phys. Lett.* B **355** 236
Khare A, MacKenzie R and Paranjape M 1995 *Phys. Lett.* B **343** 239
Chen L, Dunne G, Haller K and Lim-Lombridas E 1995 *Phys. Lett.* B **348** 468
Kao H 1998 *Phys. Rev.* D **57** 7416

[15] Brandt F T, Frenkel J and McKeon D G C 2002 *Phys. Rev.* D **65** 125029

[16] Alves V S, Das A, Dunne G V and Perez S 2002 *Phys. Rev.* D **65** 085011

[17] Gronewold H J 1946 *Physica* **12** 405
Weyl H 1949 *Z. Phys.* **46** 1
Moyal J E 1949 *Proc. Camb. Phil. Soc.* **45** 99
Doplicher S, Fredenhagem K and Roberts J E 1995 *Commun. Math. Phys.* **172** 187

[18] Douglas M R and Nekrasov N A 2001 *Rev. Mod. Phys.* **73** 977
Szabo R J 2003 *Phys. Rep.* **378** 207
Gomes M 2002 *V Proc. XI Jorge Andre Swieca Summer School* ed G Alves, O Eboli and V Rivelles (Singapore: World Scientific)
Girotti H 2003 Noncommutative quantum field theory *Preprint* hep-th/0301237

[19] Lewin L 1958 *Dilogarithms and Associated Functions* (London: Macdonald)

[20] Kapusta J I 1993 *Finite-Temperature Field Theory* (Cambridge: Cambridge University Press)

[21] Kao H-C, Lee K, Lee C and Lee T 1994 *Phys. Lett.* B **341** 181
[22] Dunne G, Lee K and Lu C 1997 *Phys. Rev. Lett.* **78** 3434
Deser S, Griguolo L and Seminara D 1997 *Phys. Rev. Lett.* **79** 1976
Deser S, Griguolo L and Seminara D 1998 *Phys. Rev.* D **57** 7444
Deser S, Griguolo L and Seminara D 1998 *Commun. Math. Phys.* **197** 443
Deser S, Griguolo L and Seminara D 2003 *Phys. Rev.* D **67** 065016
Fosco C D, Rossini G L and Schaposnik F A 1997 *Phys. Rev. Lett.* **79** 1980
Fosco C D, Rossini G L and Schaposnik F A 1997 *Phys. Rev. Lett.* **79** 4296(E)
Brandt F T, Das A and Frenkel J 2002 *Phys. Rev.* D **65** 065013
Brandt F T, Das A, Frenkel J, Pereira S and Taylor J C 2001 *Phys. Rev.* D **64** 065018
[23] Gradshteyn I S and Ryzhik M 1980 *Tables of Integrals, Series and Products* (New York: Academic)